PIANO TECHNICIANS
Journal
OFFICIAL PUBLICATION OF THE PIANO TECHNICIANS GUILD
June 2015 Vol. 58 No.6

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features
June 2015

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Tuning Accurate Equal Temperament and Ideal F3-F4 and A3-A4 Octave Sizes Using Rapid-Beating Intervals

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General Aural Tuning Procedures

In general, there are two ways that tuners set temperaments aurally. Some use *Slow-Beating Intervals (SBI)* like perfect fourths and perfect fifths to set pitches, and then may use *Rapid-Beating Intervals (RBI)* like major thirds and major sixths to check these SBIs. Conversely, some use RBIs to set pitches while using SBIs as checks.

The focus of this article will be to show how using RBIs with specific temperament sequences and SBI checks can produce highly accurate equal temperament (ET) that may not need any refining after only one pass. The sequence analyzed will be starting with A4, tuning A3 from A4, and then fitting notes between F3 and F4 inclusive.

There are basically three reasons why I am advocating this RBI Bisecting Beat Speed Window temperament sequence.

1. ET is a mathematical goal that is highly definable, and that is its beauty and power. This method considers the progressive quality of ET intervals as a standard that can be used to measure how well our ears hear beats and distinguish small differences in beat speeds, which is the essence of an aural tuner's skill.
2. Being able to set pitches with a high degree of accuracy means that, notwithstanding drifting, there should be less refining needed at the end, which should decrease tuning time.
3. It's FUN!

STEP	PROCEDURE	NOTES
The Lower Skeleton (F3-A3, A3-C♯4, C♯4-F4)		
1	Tune a beatless A3-A4.	F2-Fork = F2-A4
2	Tune F3-A3 = 7bps.	7bps is about as fast as you can tap. This may be changed later. Its accuracy is not critical.
3	Tune C♯4 so that F3-F4, A3-C♯4, and C♯4-F4 all change in beat speed by the same amount (ratio).	Depending on where you set F3, the speeds may work out as slow-medium-fast, fast-medium-slow, or medium-medium-medium. There is only one way they can work out, and it depends on where F3 was set.
The Upper Skeleton (A3-C♯4, C♯4-F4, F4-A4)		
	F3, A3, and C♯4	Tune D4 so that F3-D4 is exactly between F3-A4 and A3-C♯4.
4	Once the Lower Skeleton is set with an evenly changing beat speed ratio (increasing, decreasing, or the same, whichever works out to give smoothly and evenly changing beat speeds), tune F4 so that the Upper Skeleton is smoothly and evenly *increasing* in speed.	There were other possibilities for the Lower Skeleton, but there are no other possibilities for the Upper Skeleton. If you cannot get A3-C♯4, C♯4-F4, and F4-A4 to increase in a smooth even way, then C♯4, and/or one or both of your F3-F4 and A3-A4 octaves are not tuned correctly.
5	Re-tune F3 so that the F3-F4 octave is beatless using Figure 3.	
6	Confirm that the skeleton is tuned correctly by checking F3-A3, A3-C♯4, C♯4-F4, and F4-A4. They should all be increasing in speed by the same amount or ratio.	

Figure 1: Procedure for tuning the Skeleton.

Beat Speed Difference Ratios

When using RBIs to set chromatic ET intervals, the tuner must be able to hear small changes in beat speeds. I have defined the difference in beat speeds as the *Beat Speed Difference Ratio (BSDR)* and its formula as:

$$BSDR = (BS2 - BS1) / BS1$$

where BS1 and BS2 are beat speeds of given intervals, and BS2 > (is faster than) BS1. BSDR is expressed as a percentage.

For ET with no inharmonicity, all chromatic intervals (SBIs and RBIs) increase in beat speed by 5.95% ascending from one to the next.

Beat Speed Difference Ratio Sensitivity

From this it would appear that an aural tuner must have a BSDR sensitivity that is about half that, in order to be able to correctly judge if beat speeds of chromatic intervals are increasing, decreasing, or the same. That is, if a tuner's BSDR sensitivity was 6%, he or she would judge that a correctly tuned F3-A3 and F♯3-A♯3 beat at the same speed instead of increasing. The tuner's conclusion would be wrong.

In a limited study of six subjects, I have determined that most people, regardless of their years of aural tuning experience, have a BSDR sensitivity of 3% for rapid-beating intervals (5 to 12 beats per second, bps) and only 6% for slow-beating intervals (1 bps). This would imply that using RBIs to set accurate ET pitches should be quite possible.

The Skeleton of Contiguous Major Thirds

An RBI method that has been taught for some time for setting part of the temperament is the use of contiguous major thirds (CM3s), or what I like to call the Skeleton. This method divides the major tenth (M10) into four major thirds, and the tuner sets each beat speed to be increasing smoothly and evenly.

Here is a simple explanation of the Skeleton procedure.

- The Skeleton is F3-A3-C♯4-F4-A4.
- The Lower Skeleton is F3-A3-C♯4-F4.
- The Upper Skeleton is A3-C♯4-F4-A4.
- The Lower Skeleton is used to tune an accurate C♯4.
- The Upper Skeleton is used to tune an accurate F4 and then F3.

From analysis of ET pitches, these major thirds (M3s) increase by 25%. If a tuner can tune these CM3s by 25% +/- 3%, then the pitches will be very accurate. The problem is, a tuner can tune CM3 by 25% +/- 15% and still hear increasing beat speeds, but the pitches will not be accurate and will eventually need to be refined.

If only there was a way to gradually cut up the CM3 into finer and finer beat speed windows until we get down to the chromatic M3 at 6%…

The Inside/Outside M3/M6 Equality

One tool to help us cut up the temperament into smaller windows is the inside/outside M3/M6 equalities. Within an F3-F4 temperament octave, they are listed as

G3-B3 = F3-D4
G♯3-C4 = F♯3-D♯4
A3-C♯4 = G3-E4
A♯3-D4 = G♯3-F4

basically, a 7th chord in 3rd inversion.

With no inharmonicity consideration, the error of these assumptions is 1.87%. Given that chromatic intervals in ET increase at 5.95%, this error is acceptable.

I have determined that the second equality that we need is the minor third/major third (m3/M3) equality. This allows us to accomplish the task of creating an accurate temperament sequence that uses smaller and smaller beat speed windows and does not need any refinement.

The m3/M3 Equality

For ET with no inharmonicity, the closest m3/M3 equalities for an F3-F4 temperament octave are

F3-G♯3 = A♯3-D4
F♯3-A3 = B3-D♯4
G3-A♯3 = C4-E4
G♯3-B3 = C♯4-F4

where the error produced is also 1.87%, well within our goal of 3%. You can think of these as 7th chords in 2nd inversion.

The problem is, when inharmonicity is considered and the octave is stretched, the M3 and M6 both become wider. Fortunately, the equality is maintained within our 3% error limit. However, the m3 becomes less narrow (m3 beat speed slows down) while the M3 becomes more wide (M3 speeds up). The m3/M3 equality is destroyed.

What if we could know how much the octave is stretched using only our ears, and thereby determine the best m3/M3 equality?

This is possible using Octave Spread.

Octave Spread

I have defined Octave Spread as follows:

Octave Spread is the distance between the 6:3 coincident partials of a pure 4:2 octave. It is also the difference between the 4:2 partials of a pure 6:3 octave. And it is also the sum of the differences between the 4:2 and the 6:3 octaves for a wide 4:2, narrow 6:3. There are other combinations of octave sizes that produce other calculations for Octave Spread where both the 4:2 and 6:3 are wide, or both are narrow, or the 4:2 is narrow and the 6:3 wide. However, they all work out to the same number. Octave Spread is measured in cents.

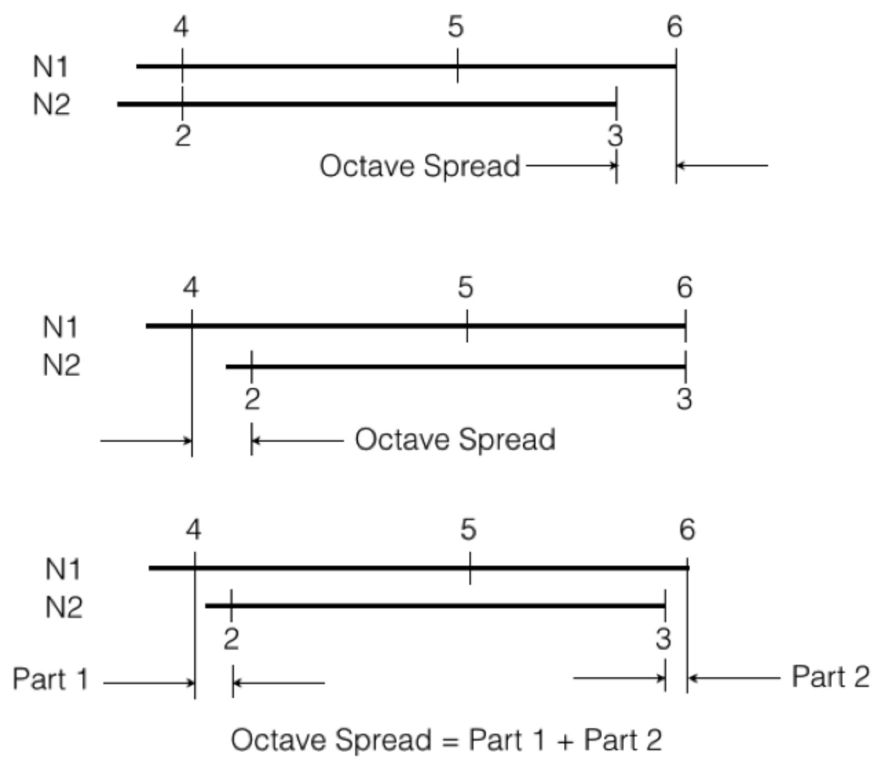

Figure 2: Graphical representation of possible octave sizes and Octave Spread for two notes, N1 and N2.

Ideal (Beatless) Octave Sizes

As an experiment, I did the following.

I tuned some F3–F4 and A3–A4 octaves so that they were as beatless as I could get them. I listened to the 4:2 and 6:3 tests and recorded them as one of the following:

1. Pure 4:2 and pure 6:3 (as far as my ear could tell)
2. Wide 4:2 and narrow 6:3.
3. Pure 4:2 with an accompanying very narrow 6:3.

All of these octaves were judged as beatless but they tested out as different 6:3 octaves.

1. Pure 6:3.
2. Narrow 6:3.
3. Very narrow 6:3.

Then I recorded each 4:2 and 6:3 test, and measured the beat speeds of each test interval: the M3 and M10 for the 4:2, and the m3 and M6 for the 6:3.

Here is what I found.

1. The limit of the ear to hear whether a M3 was different than a M10 (4:2 test), or a m3 was different than a M6 (6:3 test), was 3.6%. This is effectively the aural "pure" threshold. Any differences in the test intervals less than 3.6% will be heard as equal, and therefore the octave will test aurally as "pure" using the check intervals.

Aurally confirmed "pure" octaves are not necessarily pure, but they are below our threshold or perception. Therefore, I will describe them as "pure" with quotation marks.

This ability to measure the extent of pureness aurally is the key to tuning beatless octaves using RBIs, and choosing and using a bisecting beat speed window temperament sequence to tune highly accurate ET in one pass that does not need refining.

The previous observation helps us understand and produce beatless octaves and measure the 6:3 octave within our "pure" threshold. The next three observations help us understand and measure octave spread.

TUNING a BEATLESS OCTAVE and DETERMINING OCTAVE SPREAD

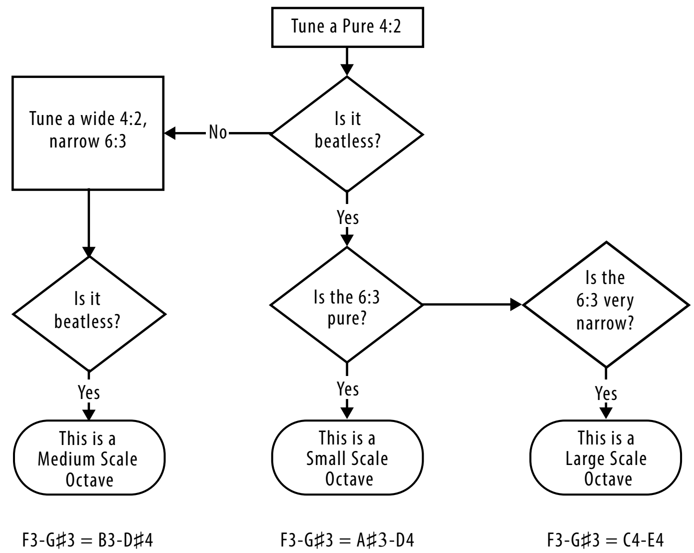

Figure 3: A method for producing beatless octaves.

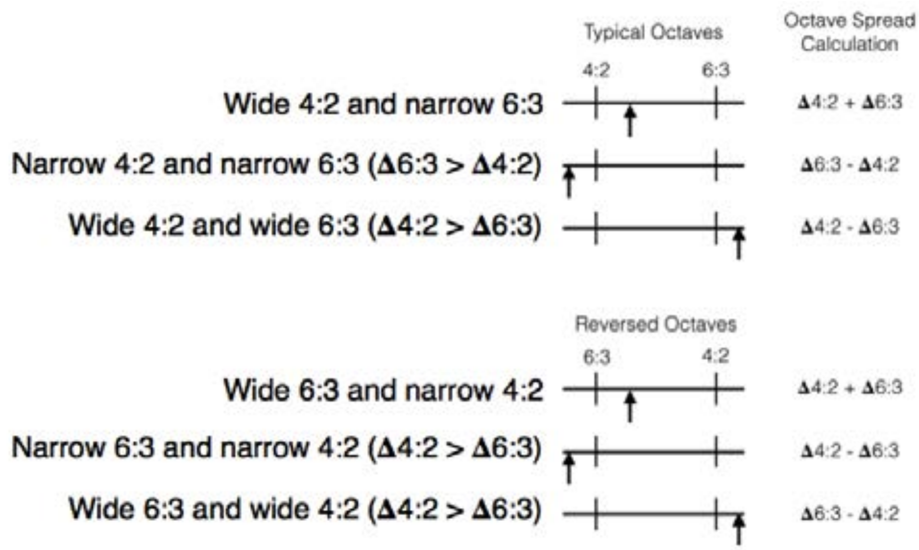

Figure 4: A graphical representation of octave sizes and typical versus reversed octaves.

2. The octave spread for the "pure" 6:3 octave was less than 0.5 cents. I defined this as a Small Spread, or Small Octave Scale.

3. The octave spread for the narrow 6:3 octave was between 0.5 and 1.1 cents, defined as a Medium Spread, or Medium Octave Scale.

4. The octave spread for the very narrow 6:3 octave (which accompanies a beatless pure 4:2 octave) was more than 1.1 cents, defined as a Large Spread, or Large Octave Scale.

Figure 3 is a flowchart method for producing a beatless octave using RBIs, which helps us to determine the octave spread, and hence the best m3/M3 equality. You need to know your 4:2 and 6:3 tests to use this chart. You also need to have a concept of what a beatless (clean, pure, or whatever you want to call it) octave sounds like.

Note that the best m3/M3 equality listed below each octave scale is valid for those intervals transposed in and around the octave being tuned. i.e. If you are tuning an A3-A4 octave and it tunes as small, then these are the possible m3/M3 equalities:

- Certainly A3-C4 = C4-A4 ("pure" 6:3) and C4-A4 = D4-F♯4 (M3/M6 equality). Therefore, A3-C4 = D4-F♯4 (another simple proof of the m3/M3 equality).
- But also, A♯3-C♯4 = D♯4-G4 and G♯3-B3 = C♯4-F4 assuming similar octave spread around the A3-A4 neighborhood.

Also, it is possible that an octave can test out as reversed (wide 6:3 and narrow 4:2) or that the F3-F4 will test out as one scale while the A3-A4 will test out as a different scale (Figure 4). There is a way to deal with these cases, but it is beyond this paper. The vast majority of cases are small-, medium-, or large-scale octaves that are typical, not reversed, and are the same for both the F and A octaves.

What Are Beatless Octaves?

The issue of beatless octaves has been debated. What I mean by "beatless" is a certain sensation that the octave is as still as possible. For medium-spread octaves, it may be that the 4:2 and the 6:3 partials are coupling because they are so close. For octaves with a large spread, it seems to my ear that the 6:3 is so far away from the 4:2 that when tuned as a wide 4:2 and a narrow 6:3, there just seems to be obvious beating at each partial, perhaps because the partials are too far away from each other to couple. But when octaves with large spread are tuned as pure 4:2, the 6:3 is beating, but perhaps so fast that it isn't as offensive. It also seems to my ear that the volume of the beating 6:3 is often quieter than a medium- or small-spread octave, and so is less offensive.

Three Possible m3/M3 Temperament Sequences Using Bisecting Beat Speed Windows

The Bisecting Window Temperament Sequences using custom m3/M3 equalities have many similar sections. Below are the three main sections.

1. Tuning of A3-A4, and the Skeleton, as well as D4 (the Set-Up).
2. Using the best m3/M3 equality to bridge to the other skeletons, or contiguous major thirds, CM3s (the Missing Links).
3. Finishing up with more CM3 bisecting sequences (the Wrap-Up).

The chart below shows the common steps with the missing m3/M3 links blank.

STEP	TUNE	USING	NOTES/PROCEDURE	AVAILABLE SBI CHECKS
	PRELIMINARY STEPS			
1	A4	Fork and F2	F2-Fork = F2-A4	A3-A4
	THE SET-UP (Skeleton plus D4)			
2	F3, A3, C♯4, and F4	See Figure 1, "Procedure for tuning the Skeleton."		
3	D4	F3, A3, and C♯4	Tune D4 so that F3-D4 is exactly between F3-A4 and A3-C♯4.	A3-D4
	The above step will be written as F3-A3 < F3-D4 < A3-C♯4. It is a bisection of the CM3, F3-A3 and A3-C♯4. All CM3 bisections will be written like this, and it is expected that the tuner will bisect the CM3 evenly by creating even beat speed ratios.			
4 & 5	The missing links here will produce accurately tuned F♯3 and A♯3.			
	The Wrap-Up that follows is common to all sequences.			
6	D♯4	F♯3-A♯3 and A♯3-D4	F♯3-A♯3 < F♯3-D♯4 < A♯3-C♯4	A♯3-D♯4
7	B3	A3-C♯4 and C♯4-F4	A3-C♯4 < B3-D♯4 < C♯4-F4	B3-F♯4
8	G3	F3-A3 and A3-C♯4	F3-A3 < G3-B3 < A3-C♯4	G3-D4
9	E4	G3-B3 and B3-D♯4	G3-B3 < G3-E4 < B3-D♯4	A3-E4
10	C4	B3-D♯4 and C♯4-F4	B3-D♯4 < C4-E4 < C♯4-F4 Already tuned for medium-octave spread sequence.	F3-C4
11	G♯3	G3-B3 and A3-C♯4	G3-B3 < G♯3-C4 < A3-C♯4 Already tuned for medium-octave spread sequence.	G♯3-D♯4, G♯3-C♯4

Figure 5: Bisecting Window Temperament Sequences.

Additional RBI Checks

At various points in the sequences there are additional RBI checks you can use. It is strongly advised to use these as well as the SBI checks listed in the charts, because the weakness in these sequences is the human ear. We need every opportunity possible to validate our judgment of beat speed differences.

Small and Large Scale Sequence

1. After step 5, check:

   F3-A3 < F♯3-A♯3 < F3-D4

   This relationship was tuned indirectly and if done correctly, it should work out. It is doesn't, re-listen to the ratios produced in steps 4 and 5.

2. After step 8, check the Whole Tone M3 Ladder: F3-A3 < G3-B3 < A3-C♯4 < B3-D♯4 < C♯4-F4

Medium Scale Sequence

1. After step 3b, check:

   F3-D4 < G♯3-C4 < A3-C♯4

   This relationship was tuned indirectly and if done correctly, it should work out. It is doesn't, re-listen to the ratios produced in steps 3a and 3b.

2. After tuning step 5 for a medium scale, check the M3 or equivalent ladder. It runs through all the M3 or equivalent intervals from F3-A3 to F4-A4.

   F3-A3 < F♯3-A♯3 < F3-D4 < G♯3-C4 < A3-C♯4 < A♯3-D4 < F3-G♯3 < F♯3-A3 < C♯4-F4 < C4-A4 < A3-C4 < A♯3-C♯4 < F4-A4

Concluding Remarks

Using beat speed windows that gradually narrow down the proper placement of pitches allows us to set pitches more precisely from the beginning of the temperament sequence, which produces reference pitches with more accuracy and should reduce tuning time.

MISSING LINK SEQUENCES USED TO TUNE F♯3 and A♯3 ACCORDING TO AURALLY DETERMINED OCTAVE SPREAD

Missing Link for the "PURE" 6:3 (Small Octave Spread)				
STEP	**TUNE**	**USING**	**NOTES/PROCEDURE**	**AVAILABLE SBI CHECKS**
4	F♯3	A3-C♯4 and C♯4-F4	A3-C♯4<F♯3-A3<C♯4-F4	F♯3-C♯4
5	A♯3	C♯4-F4 and F4-A4	C♯4-F4<A♯3-C♯4<F4-A4	F3-A♯3, A♯3-F4 F3-A♯3 = A♯3-F4

Missing Link for the NARROW 6:3 (Medium Octave Spread)				
STEP	**TUNE**	**USING**	**NOTES/PROCEDURE**	**AVAILABLE SBI CHECKS**
3a	G♯3	A3-C♯4 and C♯4-F4	A3-C♯4<F3-G♯3<C♯4-F4	G♯3-C♯4
3b	C4	C♯4-F4 and F4-A4	C♯4-F4<A3-C4<F4-A4	F3-C4, C4-F4
4	A♯3	A3-C♯4 and F3-G♯3	A3-C♯4<A♯3-D4<F3-G♯3	F3-A♯3, A♯3-F4 F3-A♯3 = A♯3-F4
5	F♯3	F3-A3 and F3-D4	F3-A3<F♯3-A♯3<F3-D4	F♯3-C♯4

Missing Link for the VERY NARROW 6:3, Pure 4:2 (Large Octave Spread)				
STEP	**TUNE**	**USING**	**NOTES/PROCEDURE**	**AVAILABLE SBI CHECKS**
4	F♯3	C♯4-F4	F♯3-A3 = C♯4-F4	F♯3-C♯4
5	A♯3	F4-A4	A♯3-C♯4 = F4-A4	F3-A♯3, A♯3-F4 F3-A♯3 = A♯3-F4

The reported size of the best octave without any other references to gauge it against is actually quite large. According to my research, many people prefer different sized octaves when asked to evaluate which octave size sounds best. However, while the window is large, I still prefer to set all octaves with similar spread as the same size, according to the criteria I laid out above, octaves which I have identified as beatless, but may be debatable. In any case, I feel the octave spread approach makes for a more uniform sound.

I would like to thank many people who helped me formulate this method, people I have never met in person and who have spent hours reading and critiquing my posts on PianoWorld.com. My thanks go to BDB, Dorrie Bell, Alfredo Capurso, Kees van denDoel, Ed Foote, Jim Ialeggio, Inlanding, David Jenson, Johnkie, Ron Koval, Chris Leslie, Gene Nelson, Isaac Olek Prout, Ryan Sowers, rXd, Chris Storch, Tunewerk, and of course, Frank Baxter, the creator of Piano World.