Unison Measurements From Beat Rates

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Recently there has been discussion of using a bandpass filter like Ocenaudio to filter beat rates of intervals.

We can also filter the beat rate of an out-of-tune unison.

By using the following method, we can calculate the cents offset of an out-of-tune unison by filtering one of its beating partials.

Procedure

1. Record an out-of-tune unison and use a bandpass filter like Ocenaudio to filter one its beating partials.

2. Drag through a number of beats in the display screen.

3. Calculate the beats per second using the formula,

$$BPS = Beats / Time$$

4. Cents is a relative measurement, not an absolute one. For a cents offset we need two frequencies. So, we will use these two frequencies:

Frequency #1 will be the partial frequency of the note given by:

$$\text{Frequency 1} = \text{Partial Number} \times \text{Equal Tempered Frequency}$$

Now, the reason why the partial is beating is because the other string’s partial is not the same frequency. Difference tone theory states that the speed of that beat is given by the difference of the frequencies.

$$\text{Beat Speed} = \text{Frequency 2} - \text{Frequency 1}, \text{ where } F2 > F1.$$

If we let Frequency 1 be the partial frequency of string 1, we can calculate the partial frequency of string 2 as follows:

$$\text{Frequency 2} = \text{Frequency 1} + \text{Beat Speed}$$

Cents offset is the same whether the 2nd frequency is higher than the 1st or lower. So, we don’t need to worry about whether the 2nd frequency is higher than the 1st or lower. This formula makes F2 arbitrarily higher.

Now that we have the two frequencies we can use the cents formula to calculate the cents offset.

$$\textbf{Cents} = 1200 \times [\ \ln(F2/F1) / \ln 2\ ]$$

ln is called the Natural Log and is found on all scientific calculators.

Proof

To prove the validity of this procedure, consider two strings, one at 440 Hz, and the other at 441 Hz.

The cents formula says that the difference between 440 Hz and 441 Hz is 3.9 cents.

Now, let us assume that we tune two A4 strings with one at 440 Hz and the other at 441 Hz.

The 3rd partial of 440 Hz is 1320 and the 3rd partial of 441 Hz is 1323. That means that the 3rd partial will be beating at 1323 - 1320 or 3 beats per second.

Another way to say this is that this third partial's beat lasts for 0.333 seconds.

Using the procedure stated, we have our two frequencies as,

F1 = equal tempered frequency of A4 x partial number
= 440 x 3
= 1320 Hz

F2 = F1 + beat rate
= 1320 + 3
= 1323 Hz

The cents formula says that the cents offset between 1320 Hz and 1323 Hz is,

$$\text{Cents} = 1200 \times [\ \ln(F2/F1) / \ln 2\ ]$$
$$= 1200 \times [\ \ln(1323/1320) / \ln 2\ ]$$
$$= 3.9 \text{ cents}$$

3.9 cents is also the cents offset between 440 Hz and 441 Hz.

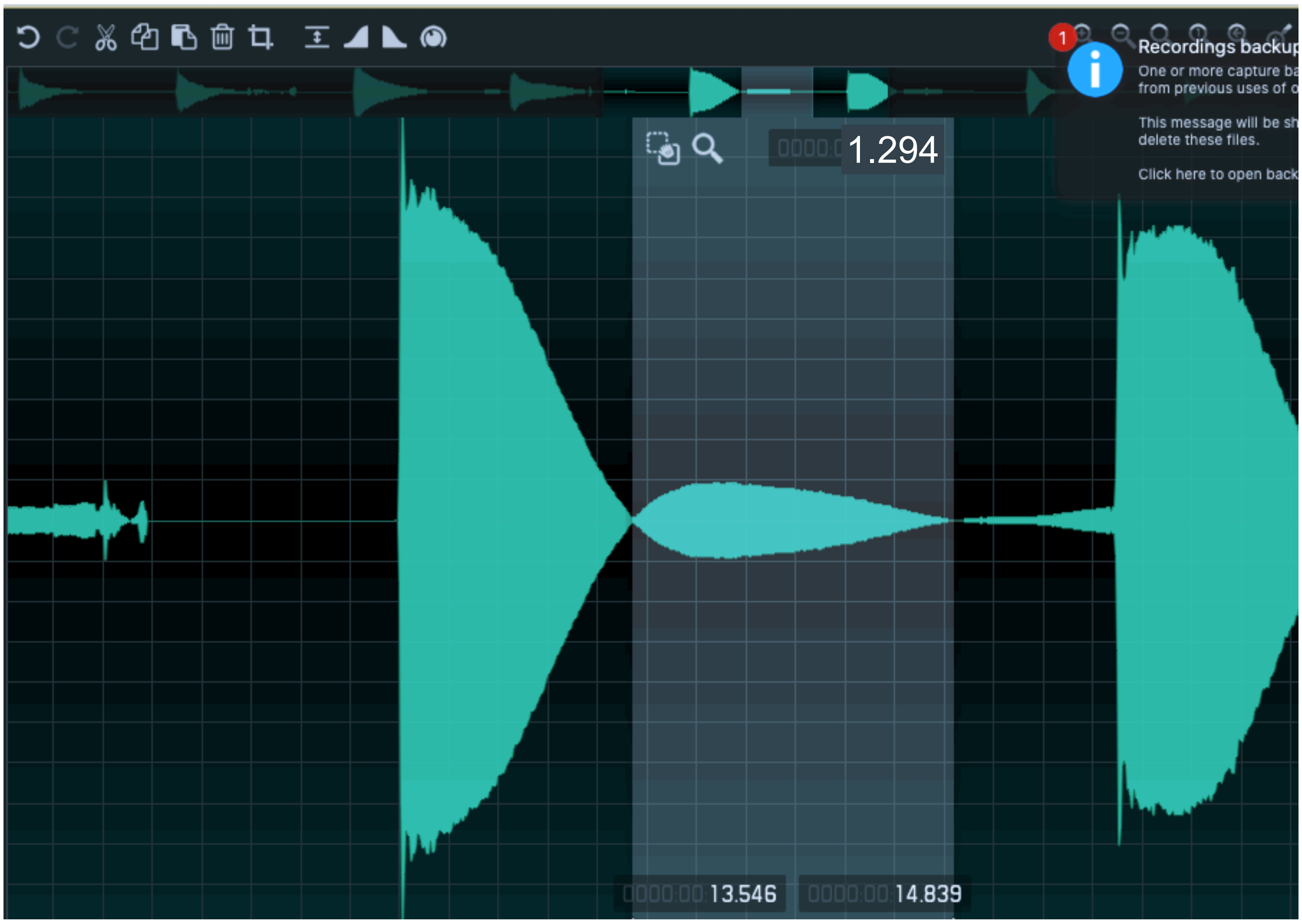
Note	A4
Number	49
Frequency	440.0
Partial	3
Beats	1
Time(x1000)	333
bps	3.0
F1	1320.0
F2	1323.0
Cents	3.9

In the spreadsheet to the left, equal tempered frequencies are calculated using the note number and the twelfth root of 2 from 440 Hz.

Example

The following charts show how an actual unison offset was calculated.

The following is a screen shot of F4 filtered around the third partial.



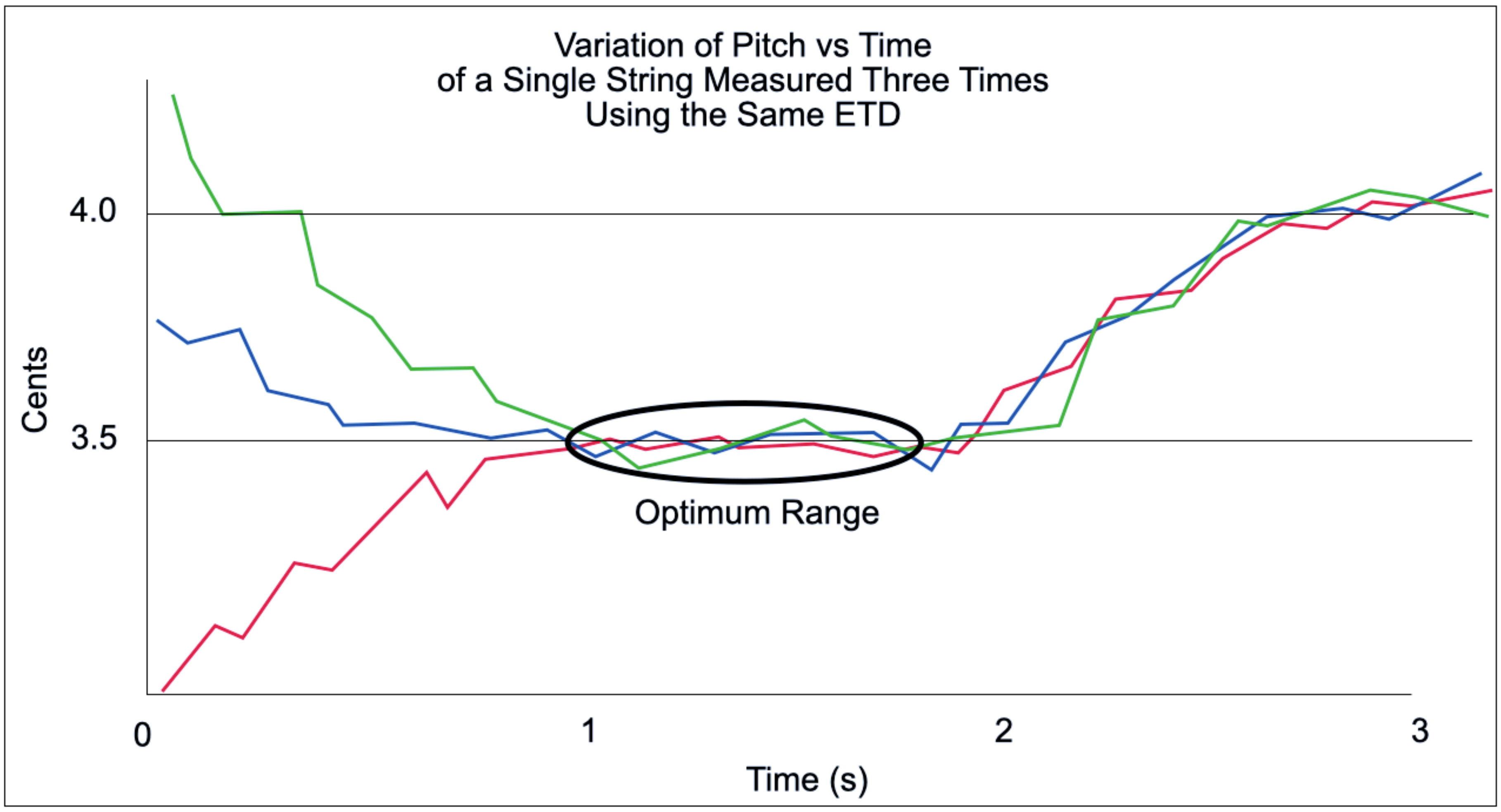
Here is a screenshot of the spreadsheet calculations.

Note	F4
Number	45
Frequency	349.2
Partial	3
Beats	1
Time(x1000)	1294
bps	0.8
F1	1047.7
F2	1048.5
Cents	1.3

Conclusion

One standard procedure for testing unisons is to use an Electronic Tuning Device (ETD) to measure each string and calculate the difference in cents and then use an arbitrary limit of 1 cent as the criteria for an acceptable unison.

The problem with this method is that:
1) The ETD takes time to settle on a pitch, and
2) that string's pitch can vary over time. See the chart below:



The actual time to settle can vary between different ETDs. The amount of drift and direction depends on the specific string's characteristics. This ETD was iStroboSoft which does not dampen the string's frequency like some other ETDs.

It is suggested that the procedure suggested in this paper is superior for the following reasons:

1) We are not measuring an unstable frequency.

2) We are not using a device that takes time to settle on a pitch.

and most importantly,

3) We are directly measuring what the examinee is trying to do, namely, eliminate higher beating partials.